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Type-I strings with F- and B-flux

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Abstract: We present non-supersymmetric toroidal compactifications of type-I string theory with both constant background NSNS two-form flux and non-trivial magnetic flux on the various D9-branes. The non-vanishing B-flux admits four-dimensional models with three generations of chiral fermions in standard model like gauge groups. Additionally, we consider the orbifold $\mathbb{T}^4/\mathbb{Z}_2$, again with both kinds of background flux present, leading to non-supersymmetric as well as supersymmetric models in six dimensions. All models have T-dual descriptions as intersecting brane worlds.

Keywords: D-branes, Superstring Vacua, Beyond Standard Model, Compactification and String Models.

Contents

1. Introduction

In type-I string theory there exist two known ways of achieving chirality in the effective lower dimensional theory. On the one hand one can compactify on curved spaces, in particular orbifolds, leading to supersymmetric [1]–[4] and non-supersymmetric [5, 6] models in six and four dimensions. On the other hand, as was first pointed out in [7] and described in a pure stringy language in [8], one can obtain chiral spectra by introducing D-branes with magnetic flux, or, in a T-dual interpretation, D-branes at angles [9]–[12, 13].

A nice geometric feature of such models is that the matter fields are localized on the intersection locus of any two branes with their multiplicity determined simply by the number of such intersection points. In general, D-branes at angles not only lead to chirality but also break supersymmetry completely at the string scale. In principle, this can be reconciled by lowering the string scale to the electroweak regime in the way of a large volume compactification. As was pointed out in [14, 15] for the case of type-II theory, such intersecting brane world models have appealing phenomenological implications, as for instance the absence of perturbative proton decay and a possible hierarchy of the Yukawa couplings. However, for D-branes at angles there generically appear tachyons in the spectrum, which in type-II theory may trigger a decay into the vacuum. For this latter reason, it is worthwhile to consider type-I models, where such a decay is forbidden due to the charge of the orientifold planes.

Because of the relatively mild tadpole cancellation conditions this ansatz leads to a plethora of solutions allowing to construct semi-realistic models in a bottom-up approach. In addition to presenting the general framework, in [8] we pointed out two subtleties for phenomenological applications of such models. First, by introducing D-branes at angles into type I it turned out to be impossible to get an odd number of matter generations due to the symmetry of the D-brane spectrum under the world sheet parity. Second, with the internal space being a torus, $\mathbb{T}^D$, a large extra dimension scenario was not compatible with chirality. This latter subtlety can be resolved by considering e.g. backgrounds of the form $\mathbb{T}^d \times \left(\mathbb{T}^{D-d}/\mathbb{Z}_3\right)$ where the D-branes wrap only $d/2$-cycles of the first torus and are pointlike on the orbifold. This will work just as in the type-II scenario of [14, 15], where, moreover, the first problem did not arise at all. Another general feature of all the non-supersymmetric vacua is, that there remains an uncancelled NSNS tadpole which leads to a shift in the background, as shown in [16].

In this paper we continue to elaborate on type-I strings with magnetic flux and resolve the first subtlety by introducing discrete NSNS two-form flux [17], as well. Via T-duality this corresponds to a tilt of the dual torus. In section 2 we explain how the non-trivial gauge background enters into the boundary states that provide the CFT description of the D-branes with constant background flux on their world volume. In section 3 we present the generalized tadpole cancellation conditions and construct a three generation left-right symmetric standard model. In the second part of the paper, section 4, we study the $\mathbb{T}^4/\mathbb{Z}_2$ orbifold [18, 19] of type I, again with both kinds of fluxes present, and derive the general form of the tadpole cancellation conditions, leading to a variety of new six-dimensional models. They display a modified pattern of the conventional reduction of the rank of the gauge group due to the B-field, revealing that the background B-field is not the reason for the reduction of the rank of the D9-brane gauge group in the first place. Actually it is better understood by noticing the multiple wrapping of the D9-branes on the torus. Note, that partial results for such models were already obtained in [20], where it was also shown that there exist non-trivial configurations preserving supersymmetry, which were related to (anti-)self dual background gauge fields.

2. D-branes on a $\mathbb{T}^2$ with background fluxes

In this section we discuss D-branes on a single two-dimensional torus in the presence of background NSNS B- and magnetic F-fluxes. In particular, we study how the data that describe the non-trivial gauge bundle on the torus, the Chern number and

Wilson lines, enter into the boundary states. This will be an essential ingredient to compute the open string scattering diagrams needed to check consistency of the type-I models in such backgrounds. Using this general approach we find a very intuitive mechanism for the reduction of the rank of the gauge group in the case of non-trivial B-flux. The actual cause is not the B-flux in the first place, but a multiple wrapping number of the D-branes.

Let us denote the coordinates by $x_{1,2}$ and the radii by $R_{1,2}$ and characterize the torus $\mathbb{T}^2$ by its Kähler and complex structures

$$T = T_1 + iT_2 = b + iR_1R_2\,, \qquad U = U_1 + iU_2 = i\frac{R_2}{R_1}\,. \tag{2.1}$$

Since U_1 is a continuous modulus of the theory, for simplicity, we are allowed to set it to zero. This choice enables us to remove the background NSNS field $B = b/T_2$ by a T-duality in the x_2 direction. The action of such a T-duality on the complex and Kähler structure is

$$T' = -1/U\,, \qquad U' = -1/T\,. \tag{2.2}$$

It also transforms a Dp-brane wrapping the entire torus into a D$(p-1)$-brane stretching along some 1-cycle. This is evident from the mapping of the boundary conditions

$$\begin{aligned} \partial_\sigma X_1 + \mathcal{F}\partial_\tau X_2 &= 0\,, \\ \partial_\sigma X_2 - \mathcal{F}\partial_\tau X_1 &= 0 \end{aligned} \tag{2.3}$$

of the Dp-brane with magnetic flux $\mathcal{F}$ into

$$\begin{aligned} \partial_\sigma\left(X_1 + \mathcal{F}X_2\right) &= 0, \\ \partial_\tau\left(X_2 - \mathcal{F}X_1\right) &= 0, \end{aligned} \tag{2.4}$$

for a D$(p-1)$-brane at an angle $\phi = \arctan(\mathcal{F})$ relative to the x_2-axis. The T-dual tori for the cases $b = 0$ and $b = 1/2$ are displayed in figure 1.

If we denote the 1-cycle wrapped by the D$(p-1)$-brane by

$$pe_1' + qe_2' \in H^1(\mathbb{T}'^2), \tag{2.5}$$

one can read off that

$$\mathcal{F} = B + F = b\frac{R_2}{R_1} + \frac{p}{q}\frac{R_2}{R_1}\,. \tag{2.6}$$

The last equation relates the wrapping quantum numbers p and q to the T-dual quantities that characterize the flux. In fact p translates into the number of times the Dp-brane wraps the entire torus and q to the first Chern number of the gauge bundle, both taking values in $H^2\left(\mathbb{T}^2\right)$. The condition that p and q are integers derives from the Dirac quantization in the "flux picture" and from the requirement to have a well defined rational wrapping of the D$(p-1)$-brane on the torus in the "angle picture".

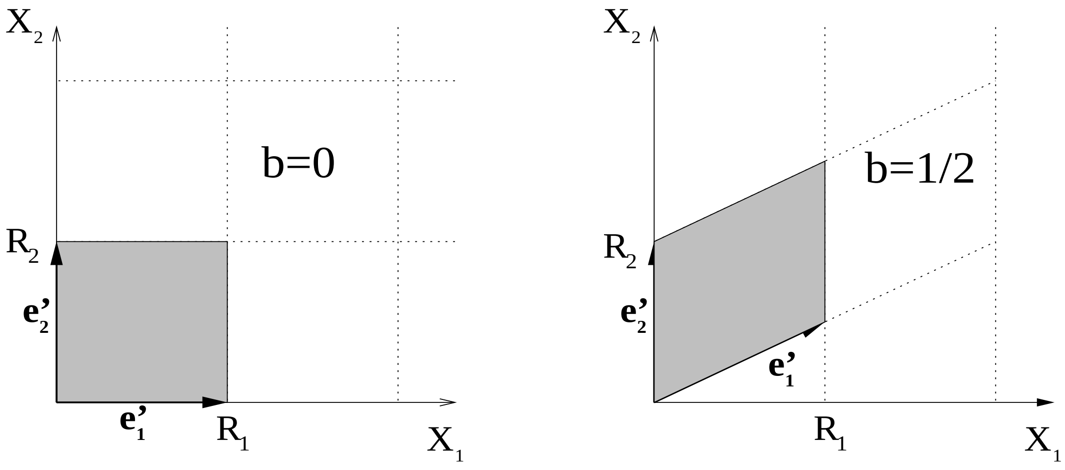

Figure 1: Dual configurations.

An interesting point to notice is that for the case $b = 1/2$ former D9-branes with $(p, q) = (2k, -k)$ now become D8-branes with even wrapping number on the cycle e'_1. Therefore, their minimal length is twice as long as compared to the case $b = 0$. Former D5-branes, which were point like on the torus now have general integer wrapping on e'_2. Therefore, a single D9-brane in the presence of the B-flux carries twice the charge of a D9-brane feeling no B-flux. Contrarily, a D5-brane still carries one unit of RR-charge. This is the origin of the reduction of the rank of the gauge group supported by the D9-branes: While the negative background charge of the O9-plane stays unchanged, D9-branes carry a double amount of charge, thus their number is halved.

In the following we will work in the "angle picture", keeping in mind how it is related to the "flux picture". The open string Kaluza-Klein and winding mass spectrum for coprime integers p and q is

$$M^2_{\mathrm{open}}(r, s) = \frac{r^2 + s^2\,(R_1 R_2)^2}{(q + b\,p)^2\,(R_2)^2 + p^2\,(R_1)^2}\,, \tag{2.7}$$

where the denominator is simply the squared length of the brane along the wrapped cycle

$$L = \sqrt{(q + b\,p)^2\,(R_2)^2 + p^2\,(R_1)^2}\,, \tag{2.8}$$

while $R_1 R_2 / L$ is the distance between two copies of the brane in the elementary cell. Since loop and tree channel are related by a modular transformation, the spectrum (2.7) affects the normalization of the boundary state of a D$(p-1)$-brane with wrapping numbers p and q

$$|B\rangle_{(p,q)} = \frac{1}{\sqrt{8}}\frac{L}{\sqrt{R_1\,R_2}}\Big(|NSNS,+\rangle - |NSNS,-\rangle + |RR,+\rangle + |RR,-\rangle\Big). \tag{2.9}$$

The non-trivial contribution of the bosonic modes along the torus $\mathbb{T}^2$ is given by

$$|B\rangle^{\rm bos}_{(p,q)} = \sum_{k,\omega} \exp\left(\sum_{n=1}^{\infty} \frac{1}{n}\, e^{2i\phi}\, \alpha_{-n}\widetilde{\alpha}_{-n} + \text{ c.c. } \right) |k,\omega\rangle, \tag{2.10}$$

with the zero-mode eigenstate $|k,\omega\rangle$ and a similar state for the fermionic modes. The dependence of the boundary state on the Chern number of the gauge bundle enters via ϕ, the normalization factor and the zero modes $|k,\omega\rangle$. Wilson lines can easily be implemented by including phase factors into the zero-mode summations. Together the boundary state encodes all the geometrical data.

Being equipped with the boundary states of wrapped branes, it is a straightforward exercise to compute the various one-loop amplitudes. In particular, transforming the tree channel annulus amplitude for two non-parallel D-branes into the loop channel reveals an extra multiplicative factor in front of the amplitude,

$$I_{\mu\nu} = p_\mu q_\nu - q_\mu p_\nu, \tag{2.11}$$

which is nothing else than the intersection number of the two D-branes on the torus $\mathbb{T}^2$. Thus, as we are used to in string theory, a pure conformal field theory computation allows us to compute topological data. Taking the intersection numbers (2.11) into account is essential when one computes the massless open string spectrum. Similarly, when considering models with an additional orbifold action, the twisted sector components of the boundary states contain all information about the number of intersection points invariant under the orbifold action.

3. Non-supersymmetric toroidal models with B- and F-flux

In this section we generalize the previous work [8] on type-I strings on a torus with magnetic background F-flux to the most general combination of F- and NSNS B-flux. To begin with, let us fix some notation. The four- or six-dimensional torus is assumed to factorize according to

$$\mathbb{T}^{2d} = \bigotimes\nolimits_{j=1}^{d} \mathbb{T}^2_{(j)} \tag{3.1}$$

with vanishing $U_1^{(j)}$ for all the two-dimensional tori. As usual, due to the Ω projection the NSNS $B^{(j)}$-field is constrained to take the discrete values $b^{(j)} = 0, 1/2$. Now, we add K stacks of N_μ D9_μ-branes with magnetic F_μ-flux turned on and look for configurations cancelling the RR tadpole of the Klein-bottle.

As explained above, this configuration is T-dual to a configuration of D$(9-d)$-branes intersecting at angles on a torus without background fluxes but with non-trivial complex structure and relative angles between the branes. Thus, in the T-dual model we have a purely geometric picture of what is going on. Note, that the world-sheet parity Ω gets transformed into $\Omega\mathcal{R}$ by this T-duality, where $\mathcal{R}$ denotes a reflection of all the $x_2^{(j)}$ directions of the $T^2_{(j)}$. Thus, for $b=0$, Ω maps a (p,q) brane to $(p,-q)$ brane, whereas for $b=1/2$ it sends a (p,q) brane to a $(p,-p-q)$ brane. Overall, we need to have $2K$ stacks of branes counting branes and their mirror branes separately.

3.1 Tadpole cancellation

We will not present a comprehensive computation of the one-loop tadpole contributions but merely state the differences as compared to the earlier work with $b=0$ in [8]. Indeed, the explicit expressions for the amplitudes on the torus can be obtained from those on $\mathbb{T}^4/\mathbb{Z}_2$, as given in the next chapter, by simply omitting the contributions arising from the orbifold insertion in the loop channel trace.

The presence of $b^{(j)}=1/2$ concretely enters the amplitudes at three distinguished points. First, the Kaluza-Klein and winding mass spectrum in (2.7), second the number of intersection points of two branes and finally the number of such intersections which are invariant under Ω. In all three cases the modification can be summarized by noting that the winding numbers $(n_\mu^{(j)}, m_\mu^{(j)})$ for the $b^{(j)}=0$ case are replaced by $(p_\mu^{(j)}, q_\mu^{(j)}+p_\mu^{(j)}/2)$. The origin of this is exclusively found in the different normalization of the boundary state due to the zero-mode spectrum (2.7). Since the Klein-bottle amplitude remains unchanged, the tadpole cancellation conditions in six dimensions read

$$\begin{aligned}\sum_{\mu=1}^{K} \mathrm{N}_\mu \prod_{j=1}^{2} p_\mu^{(j)} &= 16\,,\\ \sum_{\mu=1}^{K} \mathrm{N}_\mu \prod_{j=1}^{2} \left(q_\mu^{(j)} + b^{(j)}\, p_\mu^{(j)}\right) &= 0\,,\end{aligned} \tag{3.2}$$

while those in four dimensions are

$$\begin{aligned}\sum_{\mu=1}^{K} \mathrm{N}_\mu \prod_{j=1}^{3} p_\mu^{(j)} &= 16\,,\\ \sum_{\mu=1}^{K} \mathrm{N}_\mu\, p_\mu^{(1)} \prod_{j=2,3} \left(q_\mu^{(j)} + b^{(j)}\, p_\mu^{(j)}\right) &= 0\,,\\ \sum_{\mu=1}^{K} \mathrm{N}_\mu\, p_\mu^{(2)} \prod_{j=1,3} \left(q_\mu^{(j)} + b^{(j)}\, p_\mu^{(j)}\right) &= 0\,,\\ \sum_{\mu=1}^{K} \mathrm{N}_\mu\, p_\mu^{(3)} \prod_{j=1,2} \left(q_\mu^{(j)} + b^{(j)}\, p_\mu^{(j)}\right) &= 0\,.\end{aligned} \tag{3.3}$$

Remember that pure D9-branes without any flux correspond to D$(9-d)$-branes with even p, so that a theory with only D9-branes has a gauge group of rank $16/\mathrm{rk}(B)$. Apparently, this rank reduction is not a direct consequence of the background B-field, but only follows from the doubled wrapping number. The general spectrum of massless chiral fermions is formally the same as given in [8], with multiplicities derived from $(p_\mu^{(j)}, q_\mu^{(j)} + p_\mu^{(j)}/2)$ instead of $(n_\mu^{(j)}, m_\mu^{(j)})$.

3.2 A three generation model

In [8] we studied in a bottom-up approach toroidal models with vanishing B-field and found that the net generation number of chiral fermions was forced to be even. Thus, it was impossible to construct a standard model like spectrum with three generations.

In this section we will show that this constraint is weakened by turning on background B-form flux respectively tilting the torus in the T-dual picture. In particular, we present a four-dimensional left-right symmetric standard model with three generations.

$\mathrm{SU}(3)\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)^4$	number
$(\mathbf{3},\mathbf{2},\mathbf{1})_{(1,1,0,0)}$	2
$(\mathbf{3},\mathbf{2},\mathbf{1})_{(1,-1,0,0)}$	1
$(\overline{\mathbf{3}},\mathbf{1},\mathbf{2})_{(-1,0,1,0)}$	2
$(\overline{\mathbf{3}},\mathbf{1},\mathbf{2})_{(-1,0,-1,0)}$	1
$(\mathbf{1},\mathbf{2},\mathbf{1})_{(0,-1,0,1)}$	3
$(\mathbf{1},\mathbf{1},\mathbf{2})_{(0,0,-1,-1)}$	3

Table 1: Chiral massless spectrum

To this end, we choose four stacks of D6-branes with numbers $N_1 = 3$, $N_2 = N_3 = 2$ and $N_4 = 1$. Moreover, only on the second torus we turn on $b^{(2)} = 1/2$. The following choice of wrapping numbers

$$p_\mu^{(j)} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 3 \end{pmatrix}, \qquad q_\mu^{(j)} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & -2 & -2 \\ 1 & 0 & 0 & 1 \end{pmatrix} \tag{3.4}$$

satisfies the tadpole cancellation conditions and leads to the gauge group $\mathrm{U}(3) \times \mathrm{U}(2) \times \mathrm{U}(2) \times \mathrm{U}(1)$ and the chiral massless spectrum in table 1.

Computing the mixed $G^2 \times \mathrm{U}(1)$ anomalies, one realizes that only two of the four U(1) factors are anomaly-free. The remaining two should get a mass by a generalized Green-Schwarz mechanism involving NSNS scalars [14]. In particular

$$\mathrm{U}(1)_{B-L} = \frac{1}{3}\left(\mathrm{U}(1)_1 - 3\ \mathrm{U}(1)_4\right) \tag{3.5}$$

$\mathrm{SU}(3)\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)^2$	number
$(\mathbf{3},\mathbf{2},\mathbf{1})_{(1/3,1)}$	2
$(\mathbf{3},\mathbf{2},\mathbf{1})_{(1/3,-1)}$	1
$(\overline{\mathbf{3}},\mathbf{1},\mathbf{2})_{(-1/3,1)}$	2
$(\overline{\mathbf{3}},\mathbf{1},\mathbf{2})_{(-1/3,-1)}$	1
$(\mathbf{1},\mathbf{2},\mathbf{1})_{(-1,-1)}$	3
$(\mathbf{1},\mathbf{1},\mathbf{2})_{(1,1)}$	3

Table 2: Chiral massless spectrum.

is one of the anomaly-free abelian gauge groups. The model is designed such that the spectrum comes with the correct quantum numbers to use it as the U(1) for $B - L$. The final chiral massless spectrum is shown in table 2.

Depending on the actual values of the angles, we get tachyons in bi-fundamental representations of the gauge group, which trigger a decay of in each case two D-branes at angles into a supersymmetric 3-cycle [21, 22]. This model should serve as an example to show how easily standard model like spectra can be constructed.

4. Type-I strings on K3 with background fluxes

We now consider type-I string theory compactified to six dimensions on K3 at its $\mathbb{T}^4/\mathbb{Z}_2$ orbifold point. As in the previous section we allow the presence of constant NSNS B-flux in the bulk as well as constant magnetic F-flux on the D9-branes. Whereas partial results have already been presented in [20], our aim is to treat the problem quite generally again employing the "angle picture". The torus is defined as in the previous sections and $\mathbb{Z}_2 = \{1, \Theta\}$.

4.1 Tadpole cancellation

In the following we compute explicitly the contributions to the one-loop tadpoles. In addition to the different normalizations of the boundary states describing the D-branes, we now also have to face a different normalization of the O5-plane cross-cap state by a factor of $2^{-\mathrm{rk}(B)/2}$, which affects the Klein Bottle and the Möbius strip. In the following we present the results for the tree channel amplitudes and extract the contributions to the massless tadpoles. The Klein bottle in tree channel reads

$$\begin{aligned}\tilde{\mathcal{K}} &= \int_0^\infty dl \;\left(\langle\Omega\mathcal{R}|e^{-\pi l \mathcal{H}_{\mathrm{cl}}}|\Omega\mathcal{R}\rangle + \langle\Omega\mathcal{R}\Theta|e^{-\pi l \mathcal{H}_{\mathrm{cl}}}|\Omega\mathcal{R}\Theta\rangle\right) \\ &= 2^6\, c\,(1-1)\int_0^\infty dl \left[\frac{\vartheta[{1/2 \atop 0}]^4}{\eta^{12}}\left(\prod_{j=1}^{2}\frac{R_1^{(j)}}{R_2^{(j)}}\prod_{i,j=1}^{2}\sum_{r\in\mathbb{Z}}\exp\left(-4\pi l r^2\left(R_i^{(j)}\right)^2\right) + \right.\right. \\ &\qquad\qquad \left.\left. + \prod_{j=1}^{2}\frac{1}{16^{b^{(j)}}}\frac{R_2^{(j)}}{R_1^{(j)}}\prod_{i,j=1}^{2}\sum_{s\in\mathbb{Z}}\exp\left(\frac{-4\pi l s^2}{16^{b^{(j)}}(R_i^{(j)})^2}\right)\right)\right], \qquad (4.1)\end{aligned}$$

leading to the following contribution to the massless RR tadpole

$$\tilde{\mathcal{K}} \sim 2^{10}\left(\prod_{j=1}^{2}\frac{R_1^{(j)}}{R_2^{(j)}} + \prod_{j=1}^{2}\frac{1}{16^{b^{(j)}}}\frac{R_2^{(j)}}{R_1^{(j)}}\right)\int_0^\infty dl. \qquad (4.2)$$

As is well known, the charge of the former orientifold O9-plane state, corresponding to $|\Omega\mathcal{R}\rangle$, remains unchanged, while the charge of the former O5-planes $|\Omega\mathcal{R}\Theta\rangle$ is reduced by the B-flux.

Next we have to compute the annulus amplitude, which receives contributions from all the open strings stretching among the various D7-branes. We denote by $\gamma_{\Theta,\mu}$ the action of the orbifold generator on the Chan-Paton indices of a string ending on a D9_μ-brane. We let $\Delta_\mu^{(i)} = 0, 1$ count if the D9_μ-brane passes through the ith $\mathbb{Z}_2$ fixed point, $i = 1, \ldots, 16$. Note, that each D7-brane runs exactly through four different fixed points. The RR tree channel annulus amplitude of strings with both ends on a single D7-brane in loop channel reads

$$\begin{aligned}
\tilde{\mathcal{A}}_{\mu\mu} &= \int_0^\infty dl\ \langle D_\mu, RR|e^{-\pi l \mathcal{H}_{\rm cl}}|D_\mu, RR\rangle \\
&= 2^{-4} c \int_0^\infty dl \left[\frac{\vartheta[{1/2 \atop 0}]^4}{\eta^{12}} \mathrm{N}_\mu^2 \prod_{j=1}^2 \left(\frac{(L_\mu^{(j)})^2}{R_1^{(j)} R_2^{(j)}} \sum_{r,s\in\mathbb{Z}^2} e^{-\pi l \tilde{M}^2_{\mu,j}} \right) + \right. \\
&\qquad \left. + 4 \frac{\vartheta[{1/2 \atop 0}]^2 \vartheta[{0 \atop 0}]^2}{\eta^6 \vartheta[{0 \atop 1/2}]^2} \sum_{i=1}^{16} \Delta_\mu^{(i)} \mathrm{tr}\left(\gamma_{\Theta,\mu}\right)^2 \right] .
\end{aligned} \tag{4.3}$$

from which one can easily read off the massless untwisted and twisted tadpoles. Note that the boundary states now also include twisted components which insure the orbifold projection in the loop channel. The annulus amplitude between two different branes in tree channel is

$$\tilde{\mathcal{A}}_{\mu\nu} = \int_0^\infty dl \left(\langle D_\mu, RR|e^{-\pi l \mathcal{H}_{\rm cl}}|D_\nu, RR\rangle + \langle D_\nu, RR|e^{-\pi l \mathcal{H}_{\rm cl}}|D_\mu, RR\rangle \right) \tag{4.4}$$

$$\begin{aligned}
&= 2^{-1} c \int_0^\infty dl \left(\mathrm{N}_\mu \mathrm{N}_\nu I_{\mu\nu} \frac{\vartheta\left[{1/2 \atop 0}\right]^2 \prod_{j=1}^2 \vartheta\left[{1/2 \atop (\phi_\mu^{(j)} - \phi_\nu^{(j)})/\pi}\right]^2}{\eta^6 \prod_{j=1}^2 \vartheta\left[{1/2 \atop 1/2 + (\phi_\mu^{(j)} - \phi_\nu^{(j)})/\pi}\right]^2} + \right. \\
&\qquad \left. + \frac{\vartheta\left[{1/2 \atop 0}\right]^2 \prod_{j=1}^2 \vartheta\left[{0 \atop (\phi_\mu^{(j)} - \phi_\nu^{(j)})/\pi}\right]}{\eta^6 \prod_{j=1}^2 \vartheta\left[{0 \atop 1/2 + (\phi_\mu^{(j)} - \phi_\nu^{(j)})/\pi}\right]} \sum_{i=1}^{16} \Delta_\mu^{(i)} \Delta_\nu^{(i)} \mathrm{tr}\left(\gamma_{\Theta,\mu}\right) \mathrm{tr}\left(\gamma_{\Theta,\nu}\right) \right)
\end{aligned}$$

where the number of common fixed points can be written as

$$\sum_{i=1}^{16} \Delta_\mu^{(i)} \Delta_\nu^{(i)} = \prod_{j=1}^2 \left[1 + \frac{1}{4} \left(\sum_{\delta=0}^1 e^{\pi i \delta (p_\mu^{(j)} - p_\nu^{(j)})} \right) \left(\sum_{\epsilon=0}^1 e^{\pi i \epsilon (q_\mu^{(j)} - q_\nu^{(j)})} \right) \right] . \tag{4.5}$$

The Möbius amplitude receives only contributions from strings stretching between branes and their images under $\Omega\mathcal{R}$. In tree channel one gets

$$\begin{aligned}
\widetilde{\mathcal{M}}_\mu &= \int_0^\infty dl \left(\langle D_\mu + D_{\mu'}, RR|e^{-\pi l \mathcal{H}_{\rm cl}}|\Omega\mathcal{R} + \Omega\mathcal{R}\Theta, RR\rangle + \right. \\
&\qquad \left. + \langle \Omega\mathcal{R} + \Omega\mathcal{R}\Theta, RR|e^{-\pi l \mathcal{H}_{\rm cl}}|D_\mu + D_{\mu'}, RR\rangle \right) \\
&= \pm 2^5 c \int_0^\infty dl\, \mathrm{N}_\mu \frac{\vartheta\left[{1/2 \atop 0}\right]^2}{\eta^6} \left(\prod_{j=1}^2 \left(q_\mu^{(j)} + b^{(j)} p_\mu^{(j)} \right) \frac{\prod_{j=1}^2 \vartheta\left[{1/2 \atop -\phi_\mu^{(j)}/\pi}\right]}{\prod_{j=1}^2 \vartheta\left[{1/2 \atop 1/2 - \phi_\mu^{(j)}/\pi}\right]} + \right. \\
&\qquad \left. + \prod_{j=1}^2 p_\mu^{(j)} 4^{-b^{(j)}} \frac{\prod_{j=1}^2 \vartheta\left[{1/2 \atop 1/2 - \phi_\mu^{(j)}/\pi}\right]}{\prod_{j=1}^2 \vartheta\left[{1/2 \atop -\phi_\mu^{(j)}/\pi}\right]} \right) .
\end{aligned} \tag{4.6}$$

which leads to the following contribution to the massless RR tadpole

$$\tilde{\mathcal{M}}_\mu = \pm 2^7\, \mathrm{N}_\mu \left(\prod_{j=1}^{2} p_\mu^{(j)} \frac{R_1^{(j)}}{R_2^{(j)}} + \prod_{j=1}^{2} \left(q_\mu^{(j)} + b^{(j)} p_\mu^{(j)} \right) \frac{R_2^{(j)}}{R_1^{(j)}}\, 4^{-b^{(j)}} \right) \int_0^\infty dl. \tag{4.7}$$

Adding up all the different contributions for all possible open strings one gets the general tadpole cancellation conditions

$$\begin{aligned} &\sum_{\mu=1}^{K} \mathrm{N}_\mu \prod_{j=1}^{2} p_\mu^{(j)} = 16, \\ &\sum_{\mu=1}^{K} \mathrm{N}_\mu \prod_{j=1}^{2} \left(q_\mu^{(j)} + b^{(j)} p_\mu^{(j)} \right) = 16 \prod_{j=1}^{2} 4^{-b^{(j)}}, \\ &\sum_{\mu=1}^{2K} \Delta_\mu^{(i)}\, \mathrm{tr}\left(\gamma_{\Theta,\mu}\right) = 0, \quad i = 1, \ldots, 16. \end{aligned} \tag{4.8}$$

where in the first two equations the sum runs only over the K D7-branes without counting their mirror branes separately. In the third equation the sum runs over all $2K$ branes.

Note, that the presence of the B-flux does not necessarily imply a reduction of the rank of the gauge group. As we will show in the next subsection, we can even have $\mathrm{rk}(B) = 4$ together with a gauge group of rank 16.

4.2 Massless spectra

In this subsection we will derive the generic form of the massless spectrum distinguishing among the different intersection points. The generic solution to the twisted tadpole cancellation is simply

$$\mathrm{tr}\left(\gamma_{\Theta,\mu}\right) = 0\,, \quad \text{for all } \mu. \tag{4.9}$$

This implies a gauge group

$$\prod_{\mu=1}^{K} \mathrm{U}(N_\mu/2) \times \mathrm{U}(N_\mu/2). \tag{4.10}$$

For convenience we define $M_\mu = N_\mu/2$. The action of $\Omega\mathcal{R}$ on Chan-Paton indices is

$$\gamma_{\Omega\mathcal{R},\mu} \begin{pmatrix} A_1 & A_2 \\ A_3 & A_4 \end{pmatrix}^T \gamma_{\Omega\mathcal{R},\mu}^{-1} = \begin{pmatrix} A_4^T & A_2^T \\ A_3^T & A_1^T \end{pmatrix}, \tag{4.11}$$

the A_i being $N_\mu \times N_\mu$ matrices, A_1 for $\mu\mu$ strings, A_2 for $\mu\mu'$ strings, etc. The reflection Θ leaves all individual branes invariant and thus acts separately on the $N_\mu \times N_\mu$ factors

$$\gamma_{\Theta,\mu} \begin{pmatrix} B_1 & B_2 \\ B_3 & B_4 \end{pmatrix} \gamma_{\Theta,\mu}^{-1} = \begin{pmatrix} B_1 & -B_2 \\ -B_3 & B_4 \end{pmatrix}. \tag{4.12}$$

Sector	Spin	$(\Omega\mathcal{R}(\Theta),\Theta)$	Matter
$\mu\mu$	$(2,1)$	$(-,\Theta)$	$2((\mathbf{Adj},\mathbf{1})+(\mathbf{1},\mathbf{Adj}))$
$\mu\mu$	$(1,2)$	$(-,\Theta)$	$2(\mathbf{M}_\mu,\mathbf{M}_\mu)+\text{conj.}$
$\mu\mu'$	$(1,2)$	$(\Omega\mathcal{R}(\Theta),\Theta)$	$(\mathbf{A}_\mu,\mathbf{1})+(\mathbf{1},\mathbf{A}_\mu)+\text{conj.}$
$\mu\mu'$	$(2,1)$	$(\Omega\mathcal{R}(\Theta),\Theta)$	$(\mathbf{M}_\mu,\mathbf{M}_\mu)+\text{conj.}$
$\mu\mu'$	$(2,1),(1,2)$	$(\Omega\mathcal{R}(\Theta),-)$	$(\mathbf{A}_\mu,\mathbf{1})+(\mathbf{1},\mathbf{A}_\mu)+(\mathbf{M}_\mu,\mathbf{M}_\mu)+\text{conj.}$
$\mu\mu'$	$(2,1),(1,2)$	$(-,-)$	$(\mathbf{A}_\mu,\mathbf{1})+(\mathbf{1},\mathbf{A}_\mu)+(\mathbf{S}_\mu,\mathbf{1})+(\mathbf{1},\mathbf{S}_\mu)+$ $2(\mathbf{M}_\mu,\mathbf{M}_\mu)+\text{conj.}$
$\mu\nu,\mu\nu'$	$(1,2)$	$(-,\Theta)$	$(\mathbf{M}_\mu,\mathbf{1},\mathbf{M}_\nu,\mathbf{1})+(\mathbf{1},\mathbf{M}_\mu,\mathbf{1},\mathbf{M}_\nu)+\text{conj.}$
$\mu\nu,\mu\nu'$	$(2,1)$	$(-,\Theta)$	$(\mathbf{M}_\mu,\mathbf{1},\mathbf{1},\mathbf{M}_\nu)+(\mathbf{1},\mathbf{M}_\mu,\mathbf{M}_\nu,\mathbf{1})+\text{conj.}$
$\mu\nu,\mu\nu'$	$(1,2),(2,1)$	$(-,-)$	$(\mathbf{M}_\mu,\mathbf{1},\mathbf{M}_\nu,\mathbf{1})+(\mathbf{1},\mathbf{M}_\mu,\mathbf{1},\mathbf{M}_\nu)+$ $(\mathbf{M}_\mu,\mathbf{1},\mathbf{1},\mathbf{M}_\nu)+(\mathbf{1},\mathbf{M}_\mu,\mathbf{M}_\nu,\mathbf{1})+\text{conj.}$

Table 3: Chiral massless spectra.

Chiral massless states are localized at the intersection locus of any two D7-branes and the representation they are transforming in under the gauge group depends crucially on the behaviour of the intersection point under $\Omega\mathcal{R}$ and Θ. Moreover, a negative intersection number flips the chirality of a massless fermion. Taking all this into account the the matter content of the various open string sectors is summarized in table 3.

We always count single fermions, which, whenever the configurations become supersymmetric, may combine into proper supermultiplets. The column $(\Omega\mathcal{R}(\Theta),\Theta)$ denotes whether the intersection point is invariant under $\Omega\mathcal{R}$ or $\Omega\mathcal{R}\Theta$ and Θ, respectively, which then requires to include the proper projections.

When any two branes μ and ν pass through the same set of fixed points the conditions for the twisted tadpoles can be expressed in terms of $\mathrm{tr}\,(\gamma_{\Theta,\mu}+\gamma_{\Theta,\nu})$. In the case of $\nu=\mu'$ this results in a larger gauge group $\mathrm{U}(N_\mu)$, which happened accidently in all the examples studied in [20], where only the special cases $(p,q)\in(2,\mathbb{Z})$ of non-trivial F-fluxes in combination with B-fields have been discussed.

4.3 Examples

In this subsection we will discuss a few examples in some more detail and, as expected, we will find that all these models are anomaly free in six dimensions. The configurations of D7-branes are displayed in figure 2.

4.3.1 Example 1

We choose two stacks of branes ($K=2$) with $N_1=N_2=8$ and vanishing B-field. The following choice of wrapping numbers

$$p_\mu^{(j)}=\begin{pmatrix}1&1\\1&1\end{pmatrix},\qquad q_\mu^{(j)}=\begin{pmatrix}1&2\\2&0\end{pmatrix}\tag{4.13}$$

leads to the chiral massless spectrum shown in table 4.

sector	spin	$\mathrm{U}(4)\times\mathrm{U}(4)\times\mathrm{U}(4)\times\mathrm{U}(4)$
11,22	(2,1)	$2(\mathbf{Adj},\mathbf{1},\mathbf{1},\mathbf{1})+\text{cycl.}$
11	(1,2)	$2(\mathbf{4},\mathbf{4},\mathbf{1},\mathbf{1})+\text{conj.}$
22	(1,2)	$2(\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{4})+\text{conj.}$
11'	(1,2)	$6(\mathbf{A},\mathbf{1},\mathbf{1},\mathbf{1})+6(\mathbf{1},\mathbf{A},\mathbf{1},\mathbf{1})+2(\mathbf{4},\mathbf{4},\mathbf{1},\mathbf{1})+\text{conj.}$
22'	(1,2)	$3(\mathbf{1},\mathbf{1},\mathbf{A},\mathbf{1})+3(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{A})+1(\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{4})+\text{conj.}$
	(2,1)	$3(\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{4})+1(\mathbf{1},\mathbf{1},\mathbf{A},\mathbf{1})+1(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{A})+\text{conj.}$
12	(2,1)	$2(\mathbf{4},\mathbf{1},\mathbf{1},\mathbf{4})+2(\mathbf{1},\mathbf{4},\mathbf{4},\mathbf{1})+\text{conj.}$
12'	(1,2)	$4(\mathbf{4},\mathbf{1},\mathbf{4},\mathbf{1})+4(\mathbf{1},\mathbf{4},\mathbf{1},\mathbf{4})+2(\mathbf{4},\mathbf{1},\mathbf{1},\mathbf{4})+2(\mathbf{1},\mathbf{4},\mathbf{4},\mathbf{1})+\text{conj.}$

Table 4: Spectrum of example 1.

Together with the 20 hypermultiplets and 1 tensormultiplet from the closed string sector the spectrum in table 4 indeed satisfies F^4 and R^4 anomaly cancellation. In contrast to the toroidal case, the formal vanishing of the intersection number $I_{22'}$ does not imply a non-chiral spectrum in this open string sector due to the orbifold projection.

4.3.2 Example 2

We choose only one stack of D7-branes ($K=1$) with $N_1=N_2=16$ and wrapping numbers $(p_1,q_1)=(p_2,q_2)=(1,0)$ on the two tori. With a B-field of rank four we obtain the simple anomaly free massless spectrum in table 5.

This is an example of a model where the rank of the gauge group is not reduced by $\mathrm{rk}(B)=4$ but only by a factor of 2. Together with the 7 tensormultiplets and 14 hypermultiplets from the closed string sector the model satisfies anomaly cancellation. Note, that for appropriate choice of the radii of the two tori the flux is self-dual and the model becomes supersymmetric.

sector	spin	$\mathrm{U}(8)\times\mathrm{U}(8)$
11	(2,1)	$2(\mathbf{Adj},\mathbf{1})+2(\mathbf{1},\mathbf{Adj})$
11	(1,2)	$2(\mathbf{8},\mathbf{8})+\text{conj.}$
11'	(1,2)	$(\mathbf{A},\mathbf{1})+(\mathbf{1},\mathbf{A})+\text{conj.}$

Table 5: Spectrum of example 2.

4.3.3 Example 3

Finally, we discuss a model which was also considered in [20]. We choose two stacks of branes ($K=2$) with $N_1=N_2=4$ and non-zero B-field on the first torus $b^{(1)}=1/2$ only. A solution to the tadpole cancellation conditions is given by the wrapping numbers

$$p_\mu^{(j)}=\begin{pmatrix}2&1\\2&1\end{pmatrix},\qquad q_\mu^{(j)}=\begin{pmatrix}-1&0\\1&1\end{pmatrix},\tag{4.14}$$

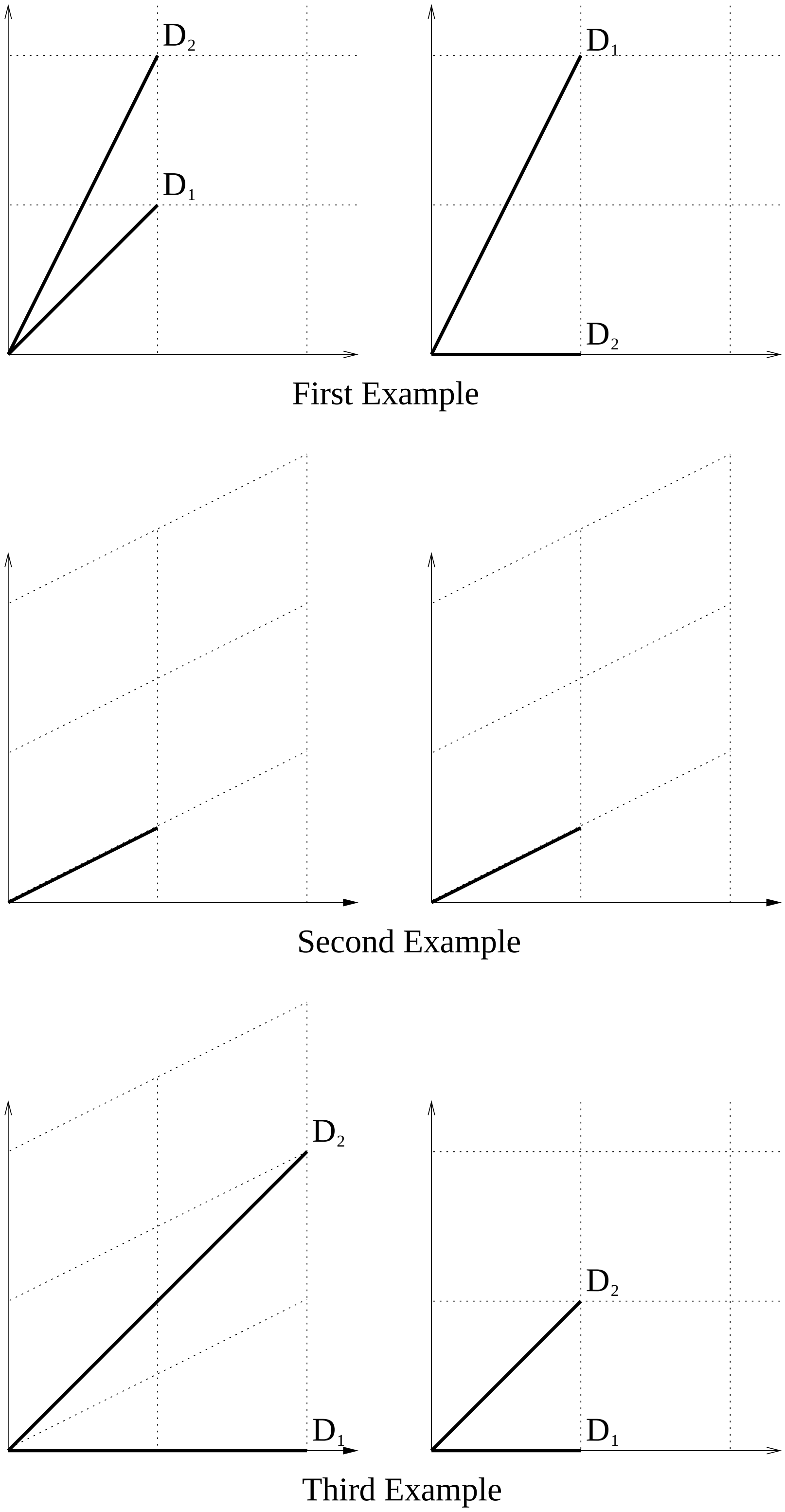

Figure 2: D7-brane configurations of 6D examples.

where the first D7-brane is stretched along the x_1 axis of the two tori, i.e. corresponds to a D9-brane without flux. Therefore, the mirror brane is of the same kind as the orginal brane and effectively we have a $U(4)$ gauge group, instead of $U(2)$. Moreover, the second brane and its mirror brane run through the same set of fixed points. Thus, we can satisfy the twisted tadpole condition by choosing

$$\mathrm{tr}\,(\gamma_{\Theta,2}) = -\mathrm{tr}\,(\gamma_{\Theta,2'})\,. \tag{4.15}$$

Therefore, there is no orbifold projection on the Chan-Paton labels for the gauge group living on the D_2-branes. Computing all the intersection numbers and taking the transformation properties of the intersection points into account we obtain the massless spectrum displayed in table 6.

sector	spin	$\mathrm{U}(4)\times\mathrm{U}(4)$
11,22	(2,1)	$2(\mathbf{Adj},\mathbf{1})+2(\mathbf{1},\mathbf{Adj})$
11	(1,2)	$2(\mathbf{A},\mathbf{1})+\mathrm{conj.}$
22'	(1,2)	$8(\mathbf{1},\mathbf{A})+2(\mathbf{1},\mathbf{S})+\mathrm{conj.}$
12	(1,2)	$4(\mathbf{4},\mathbf{4})+\mathrm{conj.}$

Table 6: Spectrum of example 3.

This completely agrees with the result obtained in [20].

5. Conclusions

In this paper we have studied type-I models with D9-branes with background B- and F-flux, respectively T-dual models with D7- or D6-branes at angles. In contrast to the case with vanishing B-flux, here we encountered no obstruction to obtain semi-realistic three generation models in a bottom-up approach. Moreover, we have studied the $\mathbb{T}^4/\mathbb{Z}_2$ orbifold model with fluxes in generality and obtained a whole plethora of anomaly-free non-supersymmetric as well as supersymmetric models in six space-time dimensions.

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